Power management of PHEV using quadratic programming

Zhiguang Zhou

College of Mechanical and Vehicle Engineering,
Hunan University,
Yuelu District, Changsha City,
Hunan Province, 410082, China
and
Department of Electrical and Computer Engineering,
University of Michigan-Dearborn,
4901 Evergreen Road, Dearborn, MI 48128, USA
E-mail: chinazhiguang@gmail.com

Chris Mi*

Department of Electrical and Computer Engineering,
University of Michigan-Dearborn,
4901 Evergreen Road, Dearborn, MI 48128, USA
E-mail: mi@ieee.org
*Corresponding author

Abstract: This paper studies the power management of a plug-in hybrid electric vehicle (PHEV) using quadratic programming. The proposed quadratic programming can obtain the global optimal solution of the power distribution between battery/motor and engine while maintaining vehicle performance. Computation time can be significantly saved compared with dynamic programming. Therefore, it can be used in real-time realisation. Two typical driving cycles, namely, UDDS and HWFET driving cycles are used to test the effectiveness of optimisation strategy. The simulation results indicate that significant amount of fuel can be saved with the proposed method.

Keywords: power management; optimisation; rule-based strategy; quadratic programming; QP; plug-in hybrid electric vehicle; PHEV; electric and hybrid vehicles.

Reference to this paper should be made as follows: Zhou, Z. and Mi, C. (2011) 'Power management of PHEV using quadratic programming', *Int. J. Electric and Hybrid Vehicles*, Vol. 3, No. 3, pp.246–258.

Biographical notes: Zhiguang Zhou received his BS in Electronic Information Engineering from Xiangtan University, China, in 2004, and his MS in Electrical Engineering from Hunan University, China, in 2007. He is a Doctoral student at the Hunan University, China. He is currently a visiting scholar in the University of Michigan-Dearborn. His research interests focus on the design, modelling, and control of hybrid electric vehicles and plug-in hybrid electric vehicles.

Chris Mi received his BSEE and MSEE in Electrical Engineering from the Northwestern Polytechnical University, Xi'an, China, and his PhD degree from the University of Toronto, Canada. He was an Electrical Engineer with General Electric Canada, Inc. He is currently an Associate Professor in the University of Michigan-Dearborn and the Director of the DTE Power Electronics Laboratory. He has been a Guest Editor of the *International Journal of Power Electronics*, a member of the editorial board of the *International Journal of Electric and Hybrid Vehicles* and the *IET Electrical Systems in Transportation*, and the Associate Editor for the *Journal of Circuits, Systems, and Computers* (2007–2009). He was the General Chair of the Fifth IEEE Vehicle Power and Propulsion Conference. He is an Associate Editor of the *IEEE Transactions on Vehicular Technology*, an Associate Editor of the *IEEE Power Electronics Letters* and a Senior Editor of the *IEEE Vehicular Technology Magazine*.

1 Introduction

It is well recognised that hybrid electric vehicle (HEV) is much more efficient and cleaner than the traditional vehicles powered by gasoline and diesel engine alone (Gao and Ehsani, 2010). However, HEVs rely on gasoline or diesel for propulsion power needs. The onboard battery provides peak-shaving function and optimises the overall fuel efficiency. Therefore, the battery state of charge (SOC) is usually maintained at the specified level over the long term of driving. Pure battery powered electric vehicle (EV) is considered as the future because it does not rely on fossil fuel. But EVs have not been accepted by the market because of its short driving range, long battery charging time and high cost (Gao and Ehsani, 2010; Ehsani et al., 2005).

Plug-in hybrid electric vehicle (PHEV) is a kind of HEV, which is purposely designed to use the on board battery to drive the vehicle using electric energy charged from the electric utility grid. Therefore, fuel can be displaced. PHEV possesses the advantages of both pure EV and HEV. PHEV can work as pure EV for certain distance when the battery SOC is high after charging, in charge-depleting (CD) operation. In CD mode, all the propulsion energy comes from battery. When battery SOC reaches a specified low level, the engine is started and the vehicle goes to charge-sustaining (CS) mode as regular HEV, during which battery SOC is maintained around this specified level until the end of trip (Gonder and Markel, 2007; Simpson, 2006; Markel and Simpson, 2006, 2005; Markel and Wipke, 2001).

Power management strategy used in PHEV is more complicated than HEV and pure EV due to the CD and CS mode. Several power management strategies, like rule-based (RB) control strategy (Gong et al., 2008), dynamic programming (DP) (Gong et al., 2008, 2007b) and stochastic dynamic programming (SDP) (Moura et al., 2010b) have been used in PHEV. RB control is easy to implement, while omitting some detailed vehicle behaviour (Gong et al., 2007a). RB is based on engineering expertise and insight, and under this control, the global optimal is absent for the whole trip. DP method obtains global optimisation for the whole trip, however, the whole driving cycle must be known at the beginning of trip. Meanwhile, the need for huge computation time makes it very difficult for real-time implementation. Nevertheless, the results from off-line computation using DP method can be used to derivation of RB control algorithms (Gao and Ehsani, 2010). For SDP method, instead of optimising over a given driving cycle, the power management strategy is optimised over a family of random driving cycles in an average sense (Lin et al., 2004). The final resolution largely dependents on the representativeness of the training driving cycles (Wu et al., 2011).

PHEVs are designed to maximise the use of battery energy during CD mode to maximise the displacement of fuel consumption. Typical control method is to use control rules that first prioritise battery energy consumption until the vehicle enters a CS mode. However, during CD operation, if the vehicle power demand is extremely high, the overall efficiency can be substantially impacted due to the large power loss in the battery. Requirement for battery power rating can also further increase battery cost. In this paper, considering the power management optimisation is a convex problem (Koot and Kessels, 2005; Moura et al., 2010a), quadratic programming (QP) method is used to obtain the optimal global resolution based on knowledge of previous trip. Compared to DP method, QP method can save computation time, which will bring prospect to the real-time implementation. Simulation results show that blended PHEV operation with the proposed QP method provides significant fuel savings over the standard CD-CS control strategy.

2 Powertrain architecture of the PHEV

In this paper, the architecture of the PHEV studied is of a power-split type similar to the Toyota Prius as shown in Figure 1 and the parameters of the PHEV is listed in Table 1. Two clutches are used to configure the operation modes of the powertrain. In this configuration, the vehicle powertrain can be configured as series operation or parallel operation. The detailed operation modes are explained as follows:

- Mode 1: motor alone mode. In this mode, C1 is engaged and C2 is open. Motor is only source of propulsion of propulsion to drive the vehicle by taking power from the battery.
- Mode 2: series mode. In series operation, C1 is engaged to ground the ring gear and C2 is open. So the engine drives the carrier which in turn transfers engine power to sun gear that drives the generator to produce electricity. The motor is the only source of propulsion. The power flow is shown in Figure 2(a).
- Mode 3: parallel mode. In parallel operation, C1 is open and C2 is engaged. So the ring gear is connected to the final drive. The planetary gear train serves as an input split device. The engine power is therefore split between sun gear and ring gear. The ring gear power is used to drive the vehicle while the sun gear power is supplied to the generator to generate electricity. The motor provide torque to drive the vehicle by taking energy from the generator or battery. The power flow of the PHEV in parallel operation is shown in Figure 2(b).

Mode transition is controlled by the transmission controller based on battery SOC and vehicle power demand. To reduce impact of mode transition on vehicle dynamic performance, C1 is always released before C2 engages.

For example, when transitioning from motor alone or series to parallel mode, engine is first started by MG2 with C1 engaged to lock the ring gear. Once engine is started, C1 can be open and the generator is controlled such that the ring gear will increase. When the ring gear speed reaches the motor speed, C2 is engaged.

When transitioning from parallel to series operation, engine power is reduced and C2 is released. At the same time, the generator is controlled such that the ring gear speed will decrease. When the ring gear speed approaches zero, C1 is engaged.

In traditional PHEV control, the vehicle will operate in the CD mode first by using the battery energy until the SOC drops below a preset threshold and at that time, the vehicle enters into CS mode. This mode is called CDCS mode. In this paper, the CD mode is defined slightly different: the engine may be started if vehicle power demand exceeds certain limit. This is due to the fact that operate the vehicle using the batter/motor only at high power demand will become very inefficient due to large losses associated with the motor and battery impedance. The modified CD mode is also called blended PHEV mode. This paper is to optimise the power distribution between the battery and engine in blended PHEV mode to minimise energy consumption (Fuel + Battery energy) for a given driving distance.

Figure 1 Architecture of PHEV

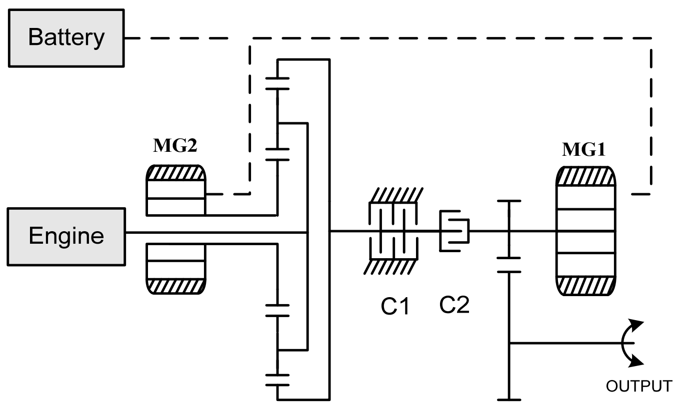


Figure 2 Power flow in the proposed of PHEV, (a) power flow in series operation (b) power flow in parallel operation

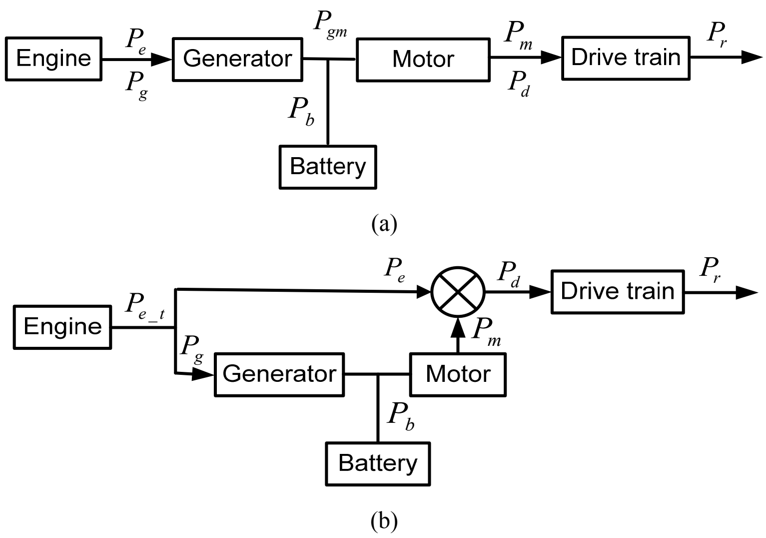


Table 1 Parameters of PHEV

Component	*Contents*	*Rating*
Vehicle	Type	SUV
	Gross weight	3,298 kg
Engine	Type	Gasoline 5.7 L
	Peak power	257 kW @ 5,300 RPM
Motor	Type	Permanent magnet AC
	Peak power	65 kW @ 4,200 RPM
Generator	Type	Permanent magnet AC
	Peak power	65 kW @ 4,200 RPM
Battery pack	Type	Lithium-ion battery
	Rated capacity	10 kWh
	Rated voltage	300 V
	Total weight	76 kg

3 Power flow analysis

From Figure 2(b), in parallel mode, the total engine power P_{e_t} is composed of two portions. One portion is P_g that is used to drive the generator to produce electricity, which can be stored in the battery (P_{gb}) or used to drive electric motor (P_{gm}) directly; another portion is P_e which is directly used to propel the vehicle, either independently, or together with power from electric motor (P_m) (Han et al., 2004). Both power from battery (P_b) and generator (P_{gm}) can be used to drive electric motor, depending on control strategy. The total engine output power can be obtained:

$$P_{e\_t} = P_e + P_g \tag{1}$$

Total power from engine has the relationship with fuel consumption:

$$P_{e\_t} = f\left(\dot{m}_f\right) \tag{2}$$

For the generator, its out power is shared between the motor and the battery, i.e.,

$$P_{gb} + P_{gm} = \eta_g \cdot P_g \tag{3}$$

where η_g is efficiency of the generator.

For the battery:

$$P_b = \eta_{b\_D} \cdot \left(\eta_{b\_C} \cdot P_{gb} + P_{bs}\right) \tag{4}$$

where η_{b_D} is discharge efficiency of battery, η_{b_C} is charge efficiency of battery and P_{bs} is the power consumed from battery.

For the motor:

$$P_m = \eta_m \cdot \left(P_b + P_{gm}\right) \tag{5}$$

where η_m is the efficiency of motor.

Output power to drive train is:

$$P_d = \eta_p \cdot \left(P_m + P_e\right) \tag{6}$$

where η_p is the efficiency of gear.

The vehicle power demand is:

$$P_r = \eta_d \cdot P_d \tag{7}$$

where η_d is the efficiency of drive train.

From equations (1) to (7), we can obtain:

$$P_r = \eta_d \eta_p \cdot \left(\eta_m \eta_{b\_D} \eta_{b\_C} P_{gb} + \eta_m P_{gm} + P_e\right) + \eta_d \eta_p \eta_m \eta_{b\_D} \cdot P_{bs} \tag{8}$$

In equation (8), the first item is the power produced by the consumption of fuel, the second item is power stored in battery, so we can rewrite (8) as:

$$P_r = g\left(\dot{m}_f\right) + \eta \cdot P_{bs} \tag{9}$$

where $\eta = \eta_d \eta_p \eta_m \eta_{b\_D}$.

From equation (9), we can see that with the previous knowledge of driving cycle, the required power is known at every time moment. So only fuel consumption and power from battery are variables, and we can use the following function to describe the relationship between fuel consumption and power from battery (Koot and Kessels, 2005; Moura et al., 2010a):

$$\dot{m}_f\left(P_{bs}\right) = \alpha_2 P_{bs}^2 + \alpha_1 P_{bs} + \alpha_0 \tag{10}$$

where $\alpha_2 > 0$ and $\alpha_0 > 0$.

4 Power management using QP

4.1 Solving the quadratic equation

From Section 3, we can see that power management problem can be described with quadratic polynomial, so the optimal solution can be obtained using QP method. Before using QP, the total time of driving cycle is discretised to n time points with the time interval Δt:

$$n = \frac{t}{\Delta t}, \quad n \in \mathbb{N} \tag{11}$$

So the relationship between fuel consumption and power from battery can be described in discrete time domain:

$$\dot{m}_f(k) = \alpha_2(k) P_{bs}^2(k) + \alpha_1(k) P_{bs}(k) + \alpha_0(k) \tag{12}$$

where $\alpha_2(k) > 0$, $\alpha_0(k) > 0$ and $k = 0, 1, ..., n$.

At every time point k, let $P_{bs}(k)$ change from:

$$\begin{cases} 0 & P_{e\_t}(k) \geq P_r(k) \\ P_r(k) - P_{e\_t\max} & P_{e\_t}(k) < P_r(k) \end{cases} \tag{13}$$

to

$$\begin{cases} P_r(k) & P_{bs\max} \geq P_r(k) \\ P_{bs\max} & P_{bs\max} < P_r(k) \end{cases} \tag{14}$$

Then corresponding fuel consumption $\dot{m}_f(k)$ can be obtained at each time point. Using least square method, coefficients $\alpha_2(k)$, $\alpha_1(k)$ and $\alpha_0(k)$ can be acquired.

The optimisation objective of power management is to minimise fuel consumption, so the cost function can be described as:

$$\begin{aligned} J &= \sum_{k=1}^{n} \dot{m}_f(k) \\ &= \sum_{k=1}^{n} \left[\alpha_2(k) P_{bs}^2(k) + \alpha_1(k) P_{bs}(k) + \alpha_0(k) \right] \end{aligned} \tag{15}$$

The target of power management is to minimise the fuel consumption under the precondition of satisfying the requirement from the driver, so it can be described as:

$$\begin{aligned} \min J &= \min\left(\sum_{k=1}^{n} \dot{m}_f(k) \right) \\ &= \min\left(\sum_{k=1}^{n} \left[\alpha_2(k) P_{bs}^2(k) + \alpha_1(k) P_{bs}(k) + \alpha_0(k) \right] \right) \end{aligned} \tag{16}$$

The QP problem can be solved analytically by introducing the Lagrange function:

$$\begin{aligned} L\left(P_{bs}(k), \lambda\right) = & \sum_{k=1}^{n} \left[\alpha_2(k) P_{bs}^2(k) + \alpha_1(k) P_{bs}(k) + \alpha_0(k) \right] \\ & - \lambda \left(\sum_{k=1}^{n} P_{bs}(k) - E_{bs} \right) \end{aligned} \tag{17}$$

where E_{bs} is the maximum usable energy supplied by battery alone during the whole trip.

The optimal solution can be obtained by solving:

$$\frac{\partial L\left(P_{bs}(k), \lambda\right)}{\partial P_{bs}(k)} = 0 \tag{18}$$

and

$$\frac{\partial L\left(P_{bs}(k), \lambda\right)}{\partial \lambda} = 0 \tag{19}$$

The solution is:

$$P_{bs\_opt}(k) = \frac{\lambda - \alpha_1(k)}{2\alpha_2(k)} \tag{20}$$

where

$$\lambda = \frac{\sum_{k=1}^{n} \frac{\alpha_1(k)}{2\alpha_2(k)} + E_{bs}}{\sum_{k=1}^{n} \frac{1}{2\alpha_2(k)}} \tag{21}$$

From equation (21), we can see that λ has the relationship with the whole driving cycle, it is to say that before using QP method, the driving cycle must be known beforehand.

4.2 Constraints of QP

In order to maintain all the components operate in the reasonable range, under the condition of driving mode, the following constraints are designed:

$$\left.\begin{array}{r} \omega_{e\_\min} \le \omega_e(k) \le \omega_{e\_\max} \\ T_{e\_\min} \le T_e(k) \le T_{e\_\max} \\ \omega_{m\_\min} \le \omega_m(k) \le \omega_{m\_\max} \\ T_{m\_\min} \le T_m(k) \le T_{m\_\max} \\ \omega_{g\_\min} \le \omega_g(k) \le \omega_{g\_\max} \\ T_{g\_\min} \le T_g(k) \le T_{g\_\max} \\ P_{bs\_\min} \le P_{bs}(k) \le P_{bs\_\max} \\ E_{bs\_\min} \le E_{bs}(k) \le E_{bs\_\max} \\ SOC_{\min} \le SOC(k) \le SOC_{\max} \end{array}\right\} \tag{22}$$

where $\omega_{e\_\min}$, $\omega_{e\_\max}$, $\omega_{m\_\min}$, $\omega_{m\_\max}$, $\omega_{g\_\min}$ and $\omega_{g\_\max}$ are the minimum and maximum speed of engine, electrical motor and generator respectively; $T_{e\_\min}$, $T_{e\_\max}$, $T_{m\_\min}$, $T_{m\_\max}$, $T_{g\_\min}$ and $T_{g\_\max}$ are the minimal and maximum torque of engine, electrical motor and generator under current speed respectively; $P_{bs\_\min}$, $P_{bs\_\max}$, $E_{bs\_\min}$ and $E_{bs\_\max}$ are the minimal and maximum power and energy from battery at the current time; and $SOC_{\min}$ and $SOC_{\max}$ are minimal and maximum SOC of battery. The energy of battery at the current time can be calculated from:

$$E_{bs}(k) = E_{bs}(0) - \sum_{k=1}^{n} P_{bs}(k) \tag{23}$$

5 Optimisation results and discussion

In order to verify the optimisation results, QP method is implemented in ADVISOR. Two typical driving cycles UDDS and HWFET are selected to test CDCS control strategy and QP method. The SOC with eight UDDS driving cycles is shown in Figure 3. It can be seen that that using CDCS method the SOC decreases quickly to the specific level 0.3 and at this time the distance is 18.6 km. On the contrary, the SOC changes slowly to the specific level of 0.3 under the control of QP, as the dashed line indicated. The simulation results under the control of CDCS and QP with different cycles of UDDS and HWFET driving cycles are shown in Figure 4 and Figure 5 respectively. In both figures, the x-axis is distance and y-axis is MPG. From Figure 4, we can see that at different distance QP method increases MPG from 3.71% to 3.97% compared to CDCS method. Following the increase of distance, MPG under both QP method and CDCS method are decreasing, but the changing rate is smaller and smaller. From Figure 5, we can see the same changing trend as Figure 4, and the same result can be obtained that using QP method has higher MPG than CDCS method, the increase is from 3.42% to 3.69%. It can also be seen that in HWFET driving cycle the MPG is higher than in UDDS driving cycle for the same distance driven.

Figure 3 SOC over the driving range for DP and CDCS (see online version for colours)

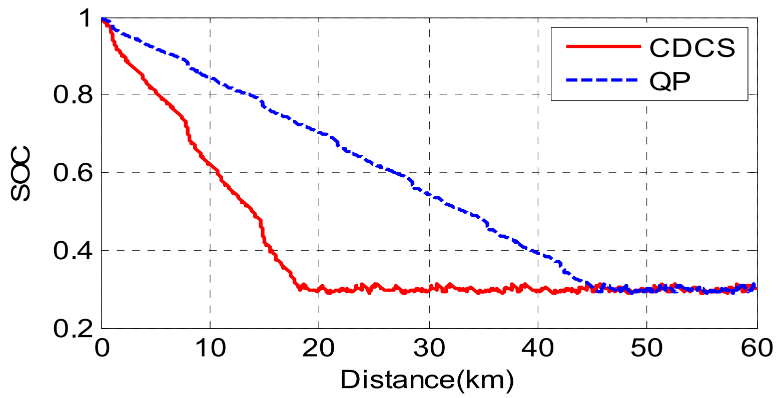


Figure 4 Fuel economy of PHEV with UDDS (see online version for colours)

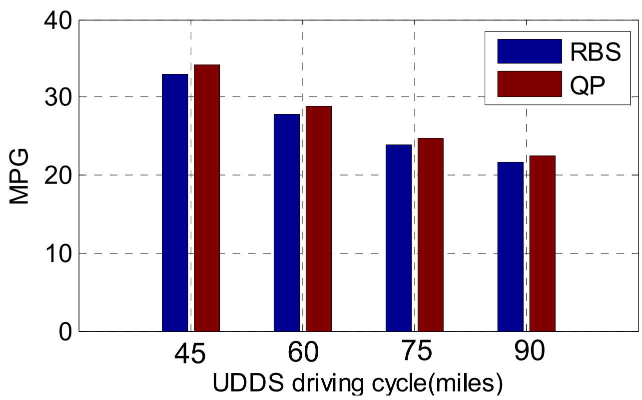
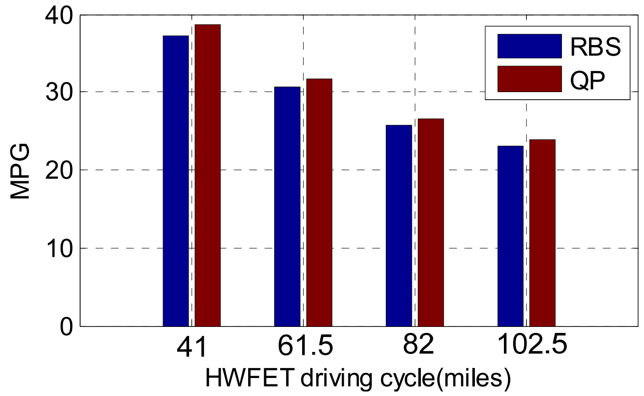


Figure 5 Fuel economy of PHEV with HWFET (see online version for colours)



The efficiency of motor and engine under the control of CDCS method and QP method in UDDS driving cycle is shown in Figure 6 to Figure 9 respectively.

From Figure 6 and Figure 7, we can see that CDCS method cannot achieve the global optimisation in the whole driving cycle, but using QP method, the global optimisation can be arrived and the motor operates in higher efficiency area compared with CDCS method. From Figure 8 and Figure 9, it can also be seen that by using QP method the engine operates in higher efficiency area compared with CDCS method. Motor and engine operating in high efficiency area will lead to the decrease of fuel consumption.

Figure 6 Efficiency of motor with CDCS method (see online version for colours)

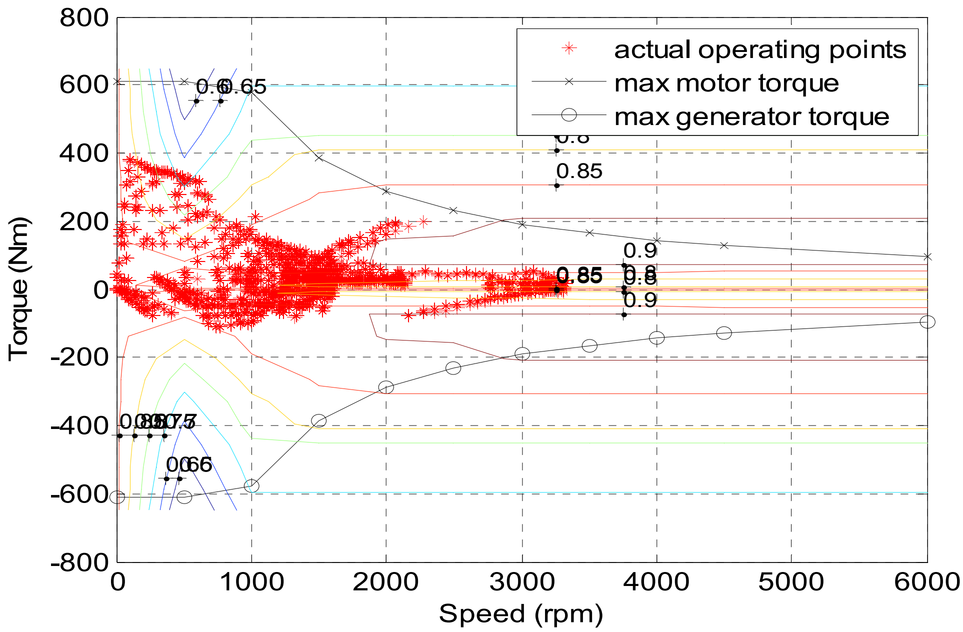


Figure 7 Efficiency of motor with QP method (see online version for colours)

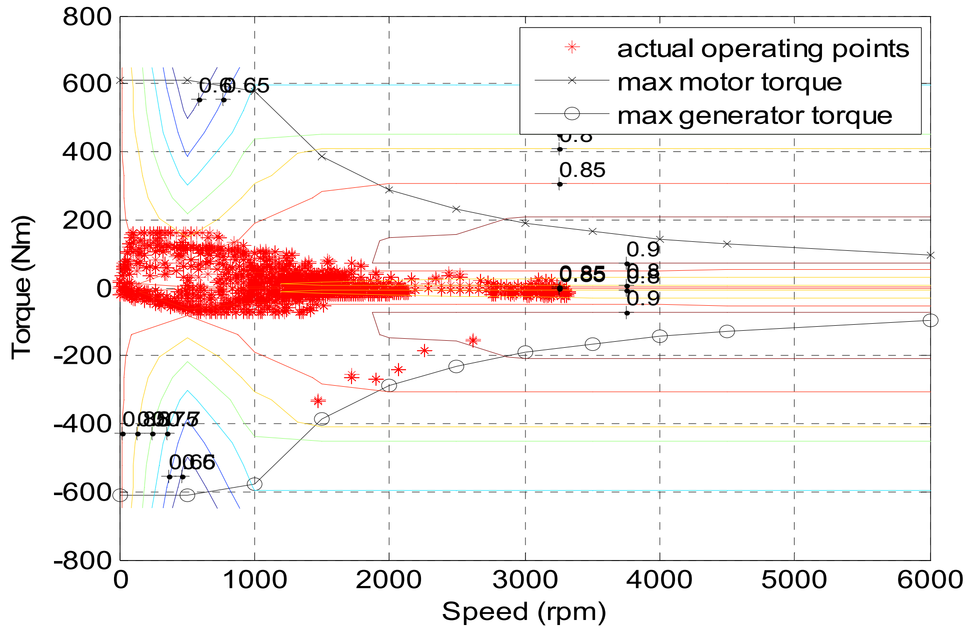


Figure 8 Efficiency of engine with CDCS method (see online version for colours)

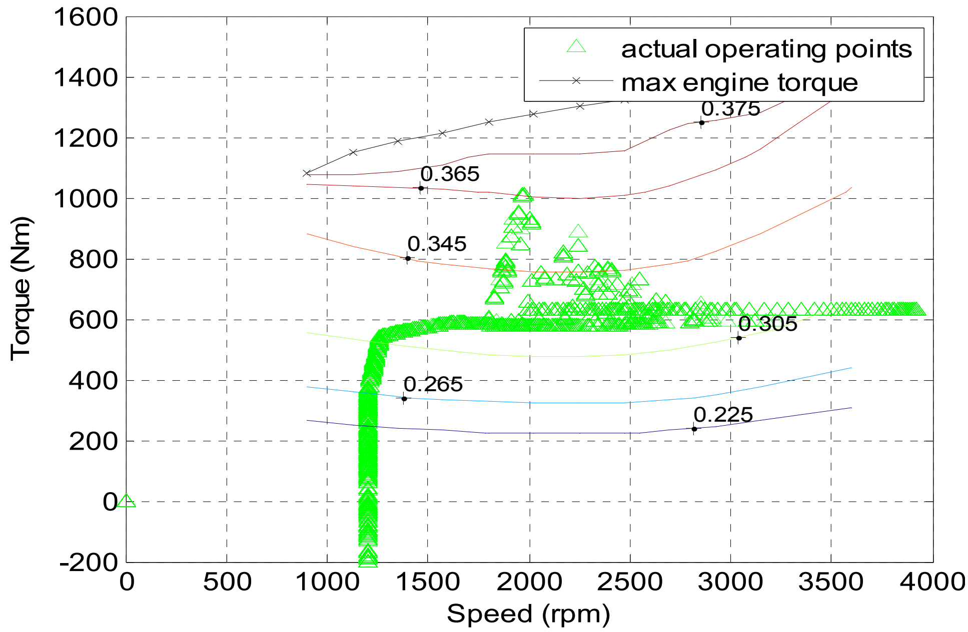
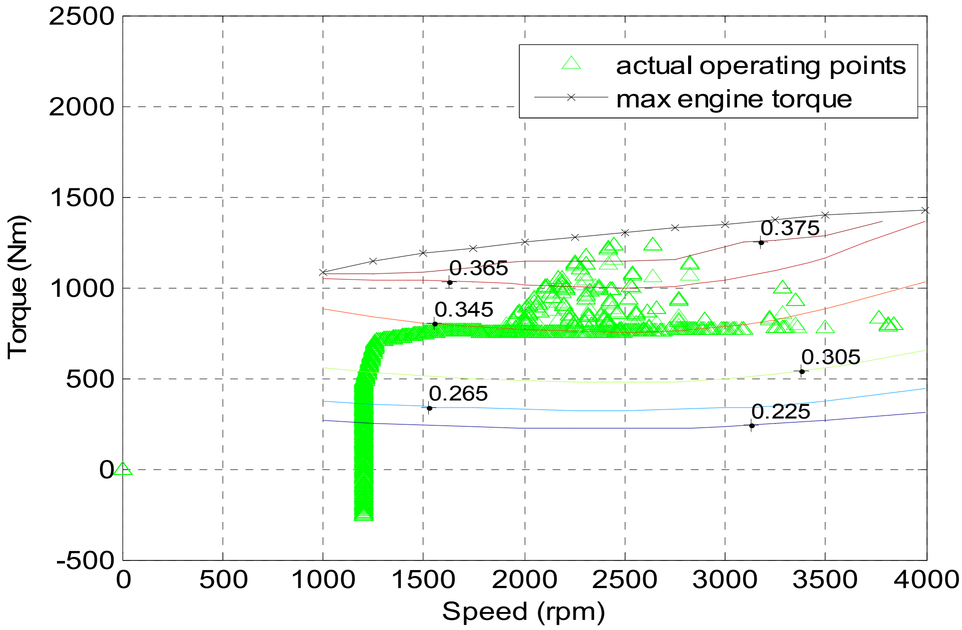


Figure 9 Efficiency of engine with QP method (see online version for colours)



6 Conclusions

In this paper, QP approach for the optimisation of PHEV power management is presented. This optimisation problem is formulated based on QP method. The variables in PHEV power management include the engine, generator, electric motor and energy storage system. The objective function is defined to minimise the fuel consumption. The satisfaction of vehicle performance requirements is the precondition of the optimisation problem and usable power range of engine and battery are selected as the constraints. The optimisation is performed in two kinds of driving cycles. The results show that in both UDDS and HWFET driving cycles, the MPG under the control of QP is higher than under the control of CDCS, it is means that QP method can decrease the fuel consumption while maintaining the vehicle performance. Furthermore, the simulation results reveal that the increase of MPG is less with the increasing of distance. The conclusion also can be drawn that with the same control method, vehicle in the HWFET driving cycle has higher MPG than in the UDDS driving cycle.

References

Ehsani, M., Gao, Y. and Gays, S.E. (2005) *Modern Electric, Hybrid Electric, and Fuel Cell Vehicle: Fundamentals, Theory, and Design*, CRC, Boca Raton.

Gao, Y. and Ehsani, M. (2010) 'Design and control methodology of plug-in hybrid electric vehicles', *IEEE Transactions on Industrial Electronics*, Vol. 57, No. 2, pp.633–640.

Gonder, J. and Markel, T. (2007) 'Energy management strategies for plug-in hybrid electric vehicles', *2007 World Congress*, Detroit, Michigan.

Gong, Q., Li, Y. and Peng, Z-R. (2007a) 'Optimal power management of plug-in HEV with intelligent transportation system', *2007 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pp.1–6.

Gong, Q., Li, Y. and Peng, Z-R. (2007b) 'Trip based power management of plug-in hybrid electric vehicle with two-scale dynamic programming', *IEEE Vehicle Power and Propulsion Conference*, pp.12–19.

Gong, Q., Li, Y. and Peng, Z-R. (2008) 'Trip-based optimal power management of plug-in hybrid electric vehicles', *IEEE Transactions on Vehicular Technology*, Vol. 57, No. 6, pp.3393–3401.

Han, Z., Yuan, Z. and Guangyu, T. (2004) 'Optimal energy management strategy for hybrid electric vehicles', *2004 SAE World Congress*, Detroit, Michigan.

Koot, M. and Kessels, J.T.B.A. (2005) 'Energy management strategies for vehicular electric power systems', *IEEE Transactions on Vehicular Technology*, Vol. 54, No. 3, pp.771–782.

Lin, C-C., Peng, H. and Grizzle, J.W. (2004) 'A stochastic control strategy for hybrid electric vehicles', *Proceedings of the 2004 American Control Conference*, 30 June to 2 July, pp.4710–4715.

Markel, T. and Simpson, A. (2005) 'Energy storage systems considerations for grid-charged hybrid electric vehicles', *SAE Future Transportation Technology and IEEE Vehicle Power and Propulsion Joint Conferences*, Chicago, Illinois.

Markel, T. and Simpson, A. (2006) 'Plug-in hybrid electric vehicle energy storage system design', *Advanced Automotive Battery Conference*, Baltimore, Maryland.

Markel, T. and Wipke, K. (2001) 'Modeling grid-connected hybrid electric vehicle using ADVISOR', *16th Annual Battery Conference on Applications and Advances*, Long Beach, California, 9–12 January, pp.23–29.

Moura, S.J., Callawayb, D.S. and Fathya, H.K. (2010a) 'Tradeoffs between battery energy capacity and stochastic optimal power management in plug-in hybrid electric vehicles', *Journal of Power Sources*, Vol. 195, No. 9, pp.2979–2988.

Moura, S.J., Fathy, H.K. and Callaway, D.S. (2010b) 'A stochastic optimal control approach for power management in plug-in hybrid electric vehicles', *IEEE Transactions on Control Systems Technology*, Vol. 19, No. 3, pp.545–555.

Simpson, A. (2006) 'Cost-benefit analysis of plug-in hybrid electric vehicle technology', *22nd International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium and Exhibition (EVS-22)*, Yokohama, Japan, pp.1–15.

Wu, X., Cao, B. and Li, X. (2011) 'Component sizing optimization of plug-in hybrid electric vehicles', *Applied Energy*, Vol. 88, No. 3, pp.799–804.